

Delicate interplay between the $D^0 D^{*0}$, $\rho^0 J/\psi$, and $\omega J/\psi$ channels in the $X(3872)$ resonance

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Abstract. The nature of the $X(3872)$ enhancement is analysed in the framework of the Resonance-Spectrum Expansion, by studying it as a regular $J^{PC} = 1^{++}$ charmonium state, though strongly influenced and shifted by open-charm decay channels. The observed but Okubo-Zweig-Iizuka-forbidden $\rho^0 J/\psi$ and $\omega J/\psi$ channels are coupled as well, but effectively smeared out by using complex ρ^0 and ω masses, in order to account for their physical widths, followed by a rigorous algebraic procedure to restore unitarity. A very delicate interplay between the $D^0 D^{*0}$, $\rho^0 J/\psi$, and $\omega J/\psi$ channels is observed. The data clearly suggest that the $X(3872)$ is a very narrow axial-vector $c\bar{c}$ resonance, with a pole at or slightly below the $D^0 D^{*0}$ threshold.

1 Introduction

The $X(3872)$ charmonium-like state was discovered in 2003 by the Belle Collaboration [1], as a $\pi^+ \pi^- J/\psi$ enhancement in the decay $B^\pm \rightarrow K^\pm \pi^+ \pi^- J/\psi$. The same structure was then observed, again in $\pi^+ \pi^- J/\psi$, by CDF II [2], D0 [3], and BABAR [4]. Moreover, CDF [5] showed that the $\pi^+ \pi^-$ mass distribution favors decays via a ρ^0 resonance, implying positive C -parity for the $X(3872)$. The $X(3872)$ has also been observed in the $\bar{D}^0 D^0 \pi^0$ and $\bar{D}^{*0} D^{*0}$ channels, by Belle [6] and BABAR [7], respectively. More recently, CDF [8] measured the $X(3872)$ mass with even higher precision, viz. $3871.61 \pm 0.16 \pm 0.19$ MeV, with a width fixed at 1.34 ± 0.64 MeV, while BABAR [9] presented evidence for the long-awaited $\omega J/\psi$ decay mode (also see Ref. [10]), and a surprising preference for the 2^{-+} assignment. We shall come back to the latter claim below. The $X(3872)$ resonance is listed in the 2010 PDG tables [11], with a mass of 3871.56 ± 0.22 MeV, a width < 2.3 MeV, and 1^{++} or 2^{-+} quantum numbers [11, 12].

On the theoretical side, the first to foresee a narrow 1^{++} state close to the DD^* threshold was Törnqvist [13], arguing on the basis of strongly attractive one-pion exchange for S -wave meson-meson systems, which he called deusons. For further molecular descriptions and studies, see Ref. [14], as well as the reviews by Swanson [15] and Klempt & Zaitsev [16]. In Ref. [17], a few exotic model descriptions can be found, such as a hybrid or a tetraquark; also see the reviews [15, 16]. For further reading, we recommend the very instructive analyses by Bugg [18] and Kalashnikova & Nefediev [19].

Much more in the spirit of our own calculation is the coupled-channel analysis by Danilkin and Simonov [20], which studies resonances and level shifts of conventional charmonium states due to the most important open and closed decay channels. We shall come back to their results below.

As said above, according to the PDG [11], the $X(3872)$ is either a 1^{++} or a 2^{-+} state, which implies 2^3P_1 or 1^1D_2 , as other radial excitations would be much too far off (see e.g. Ref. [21]). In the present paper, we only study the 1^{++} scenario, despite the conclusion by BABAR [9], from the $\omega J/\psi$ mode, that 2^{-+} is more likely. However, the latter assignment appears to be at odds with radiative-transition data [22]. For a further discussion of electromagnetic decays, see e.g. the molecular description of Ref. [23]. But more importantly, in all charmonium models we know of, the 1^1D_2 $c\bar{c}$ state lies well below 3.872 GeV, i.e., in the range 3.79–3.84 GeV (see e.g. Ref. [25]). Our own *bare* 1^1D_2 state comes out at 3.79 GeV, just as the corresponding single-channel state in Ref. [20]. Now, the crucial point is that loops from closed meson-meson channels are *always* attractive [24]. Hence, since DD^* at 3.872–3.880 GeV is the lowest Okubo-Zweig-Iizuka-allowed (OZIA) channel that couples to a 1^1D_2 $c\bar{c}$ state, the coupled-channel mass shift will inexorably be further downwards (also see Ref. [25]).

The goal of the present paper is to show that the mass and width of the $X(3872)$, as well as the corresponding observed amplitudes in the $D^0 D^{*0}$, $\rho^0 J/\psi$, and $\omega J/\psi$ channels, are compatible with a description in terms of a regular 2^3P_1 charmonium state, though mass-shifted and unitarised via open and closed decay channels.

2 Resonance-Spectrum Expansion

Sticking to the 1^{++} scenario, we employ again the Resonance-Spectrum Expansion (RSE) [26], in order to couple one $c\bar{c}$ channel, with $l_c = 1$, to several OZIA pseudoscalar-vector (PV) and vector-vector (VV) channels, just as in our recent preliminary study [27] of the $X(3872)$. However, we now also couple the OZI-forbidden (OZIF) $\rho^0 J/\psi$ and $\omega J/\psi$ channels, to account for the bulk of the observed $\pi^+\pi^- J/\psi$ and $\pi^+\pi^-\pi^0 J/\psi$ decays, respectively. Although the former channel is isospin breaking as well, the extreme closeness of its central threshold at 3872.4 MeV to the $X(3872)$ structure makes it absolutely nonnegligible, despite a very small expected coupling. A complication, though, is the large ρ width, which does not allow the $\rho^0 J/\psi$ channel to be described through a sharp threshold. Effects in the $X(3872)$ from non-zero ρ and ω widths were already estimated in Ref. [28]. We tackle this problem by taking a complex mass for the ρ , from its pole position [29], and then apply a novel, empirical yet rigorous, unitarisation procedure to the S -matrix, derived in Appendix B. The analyticity and causality implications of complex masses in asymptotic states were already studied a long time ago [30].

For consistency, we apply the same procedure to the ω meson, despite the fact that its width is a factor 17.5 smaller than that of the ρ . Nevertheless, the ω width of about 8.5 MeV is very close to the energy difference between the $\omega J/\psi$ and $D^0 D^{*0}$ thresholds, and therefore not negligible. Finally, we shall neglect the unknown small (< 2.1 MeV [11]) D^{*0} width [19], because of the relatively large error bars on the $D^0 D^{*0}$ data, though this width may have some influence on the precise $X(3872)$ pole position. Nevertheless, reasonable estimates of the D^{*0} width yield values clearly smaller than 100 keV [31], so that its effect should be largely negligible as compared to that of the ρ and ω widths.

Let us now proceed with our RSE calculation of a bare 2^3P_1 (with $n = 1$, $J = 1$, $L = 1$, $S = 1$) $c\bar{c}$ state, coupled to a number of MM channels. The resulting closed-form T -matrix is given in Appendix A. In Table 1 we list the considered PV and VV channels, including $\rho^0 J/\psi$

and $\omega J/\psi$. Besides the latter two OZIF channels and the also observed OZIA $D^0 D^{*0}$ channel, we furthermore account for the OZIA PV and VV channels $D^\pm D^{*\mp}$, $D^* D^*$, and $D_s D_s^*$, whose influence on the $X(3872)$ pole position is not negligible, in spite of being closed channels. The $D_s^* D_s^*$ channel, with threshold about 350 MeV above the $X(3872)$ mass, we do not include.

The relative couplings of the OZIA channels have been computed with the formalism of Ref. [32]. Couplings calculated in the latter scheme for ground-state mesons generally coincide with the usual recouplings of spin, isospin, and orbital angular momentum. Moreover, for excited states the formalism yields clear predictions as well, contrary to other approaches. In Table 1, the squares of the ground-state ($n=0$) couplings are given, which have to be multiplied by $(n+1)/4^n$ for the S -wave PV channels, and by $(2n/5+1)/4^n$ for the others, so as to obtain the couplings in the RSE sum of Eq. (3). Also note that the two (closed) $D^* D^*$ channels have been lumped together, with their average threshold value and sum of squared couplings. As for the couplings of the $\rho^0 J/\psi$ and $\omega J/\psi$ channels, the formalism of Ref. [32], based on OZIA decay via 3P_0 quark-pair creation, cannot make any prediction. However, we know from experiment that the couplings of OZIF channels are considerably smaller than those of OZIA channels. Moreover, isospin-breaking channels are even further suppressed. Thus, in the following we shall employ the values $g_{\rho^0 J/\psi} = 0.07 \times g_{D^0 D^{*0}}$ and $g_{\omega J/\psi} = 0.21 \times g_{D^0 D^{*0}}$, which correspond to effective relative strengths of 0.49% and 4.41%, respectively, which seem reasonable to us. These values may also be compared to the corresponding relative probabilities of about 0.65% ($\approx 0.006/0.92$) and 4.5% ($\approx 0.041/0.92$), respectively, employed in Ref. [23]. Furthermore, we shall also test coupling values twice as large, namely $g_{\rho^0 J/\psi} = 0.14 \times g_{D^0 D^{*0}}$ and $g_{\omega J/\psi} = 0.42 \times g_{D^0 D^{*0}}$. Note that our coupling for the isospin-breaking channel $\rho^0 J/\psi$ is also in rough agreement with estimates from the rate of the observed [11] isospin-violating $\omega \rightarrow \pi^+\pi^-\pi^0$ decay, which amounts to about 1.5% of the total width. Another difference between OZIA and OZIF channels is the average distance r_i (see Eqs. (1,2)) at which a light $q\bar{q}$ pair is created before decay, which in the OZIA case we believe to take place in the core region and in the OZIF case more in the periphery. Thus, we employ a larger value for $r_1 \equiv r_{\rho^0 J/\psi} = r_{\omega J/\psi}$ than for r_0 , the single radius used for all OZIA channels. Concretely, we take $r_0 = 2 \text{ GeV}^{-1} \simeq 0.4 \text{ fm}$ and $r_1 = 3 \text{ GeV}^{-1} \simeq 0.6 \text{ fm}$, while we also test the case $r_1 = r_0$.

For the bare $c\bar{c}$ energy levels $E_n^{(l_c)}$ in the RSE sum of Eq. (3), we take an equidistant harmonic oscillator (HO), as in previous work (see e.g. Ref. [33]). The corresponding charm quark mass and HO frequency are once again kept fixed at the values $m_c = 1562 \text{ MeV}$ and $\omega = 190 \text{ MeV}$. The only parameter we adjust freely is the overall coupling constant λ in Eqs. (1,2), which is tuned to move the bare 2^3P_1 state from 3979 MeV down to the $D^0 D^{*0}$ threshold, requiring a λ value of the order of 3, i.e., not far from the values used in e.g. Refs. [33,34]. At the same time, the bare 1^3P_1 state shifts from 3599 MeV down to about 3.55 GeV,

Table 1. Included meson-meson channels, with thresholds and ground-state couplings. For simplicity, we omit the bars over the anti-charm mesons; also note that $D^* D^*$ stands for the corresponding mass-averaged charged and uncharged channels.

Channel	$(g_{(l_c=1,n=0)}^i)^2$	L	Threshold (MeV)
$\rho^0 J/\psi$	variable	0	$3872.406 - i 74.7$
$\omega J/\psi$	variable	0	$3879.566 - i 4.25$
$D^0 D^{*0}$	1/54	0	3871.81
$D^0 D^{*0}$	5/216	2	3871.81
$D^\pm D^{*\mp}$	1/54	0	3879.84
$D^\pm D^{*\mp}$	5/216	2	3879.84
$D^* D^*$	5/36	2	4017.24
$D_s^\pm D_s^{*\mp}$	1/54	0	4080.77
$D_s^\pm D_s^{*\mp}$	5/216	2	4080.77

though depending quite sensitively on the precise form of the used subthreshold suppression of closed channels [33]. Anyhow, for the purpose of the present study, an accurate reproduction of the $\chi_{c1}(1P)$ mass of 3511 MeV is not very relevant.

3 $X(3872)$ poles and amplitudes vs. data

In Table 2, we give some pole positions in the vicinity of the $D^0 D^{*0}$ and $\rho^0 J/\psi$ thresholds, with the chosen values

Table 2. Pole positions of the dots and stars in Fig. 1. In all cases, $r_1 = 3.0 \text{ GeV}^{-1}$. Note that the OZIF couplings $\tilde{g}_{\rho^0 J/\psi}$ and $\tilde{g}_{\omega J/\psi}$ are given relative to the coupling of the OZIA $D^0 D^{*0}$ channel.

Label	λ	$\tilde{g}_{\rho^0 J/\psi}$	$\tilde{g}_{\omega J/\psi}$	Pole (MeV)
1	3.028	0.07	0.21	$3872.30 - i 0.71$
2	3.066	0.07	0.21	$3871.83 - i 0.40$
3	3.083	0.07	0.21	$3871.56 - i 0.11$
a	2.981	0.14	0.42	$3872.30 - i 0.75$
b	3.017	0.14	0.42	$3871.82 - i 0.48$
c	3.033	0.14	0.42	$3871.57 - i 0.28$

of λ and r_1 . In Fig. 1, third-sheet pole trajectories in the

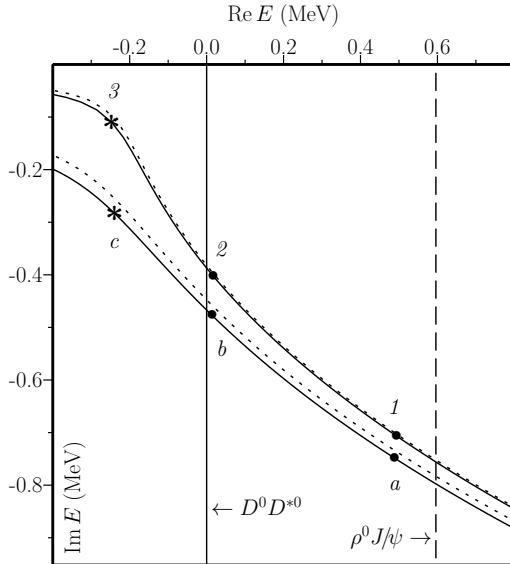


Fig. 1. Pole trajectories for $r_1=3.0 \text{ GeV}^{-1}$ (solid curves) and $r_1=2.0 \text{ GeV}^{-1}$ (dotted curves); $g_{\rho^0 J/\psi}/g_{D^0 D^{*0}} = 0.07$ and $g_{\omega J/\psi}/g_{D^0 D^{*0}} = 0.21$ (upper two curves), $g_{\rho^0 J/\psi}/g_{D^0 D^{*0}} = 0.14$ and $g_{\omega J/\psi}/g_{D^0 D^{*0}} = 0.42$ (lower two curves). Note that the CM energy E is relative to the $D^0 D^{*0}$ threshold in all figures. Also see Table 2.

complex energy plane (relative to the $D^0 D^{*0}$ threshold) are plotted, as a function of λ , with the pole positions

of Table 2 marked by bullets and stars. The solid curves represent the case $r_1 = 3.0 \text{ GeV}^{-1}$, while the dotted ones stand for $r_1 = 2.0 \text{ GeV}^{-1}$, showing little sensitivity to the precise decay radius. Figure 1 shows that the $X(3872)$ resonance pole may come out below the $D^0 D^{*0}$ threshold with a nonvanishing width, which is more over of the right order of magnitude, viz. $< 1 \text{ MeV}$. The recent CDF [8] mass determination of the $X(3872)$ might suggest that the pole positions '3' or 'c' (see Table 2 and Fig. 1) are favored. However, one should realise that the differences amount to mere fractions of an MeV, while experimental uncertainties are at least of the same order.

Now we compare the corresponding $D^0 D^{*0}$ amplitudes to Belle [35] data, for the six cases labeled '1, 2, 3', and 'a, b, c' in Table 2 and Fig. 1. The results are depicted in Figs. 2 and 3, respectively. Note that we allow for an arbitrary

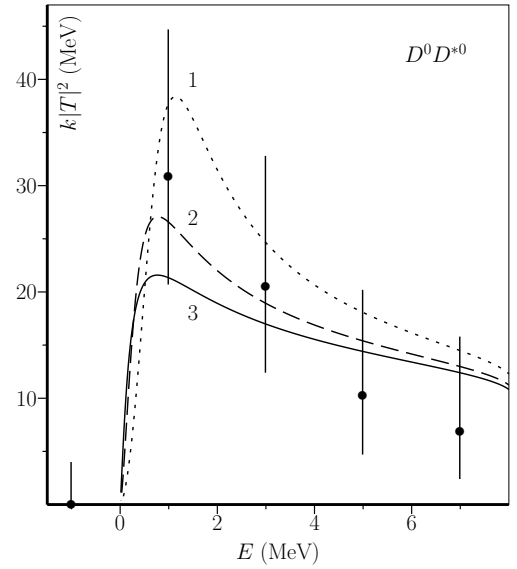


Fig. 2. $D^0 D^{*0}$ elastic amplitude for poles 1, 2, 3 in Table 2 and Fig. 1; arbitrarily normalised data are from Ref. [35]. Elastic T -matrix elements follow from Eqs. (1–3); k is on-shell relative momentum.

normalisation of the data, which is inevitable as we are dealing with production data, which cannot be directly compared with our scattering amplitudes, also because of the finite experimental mass bins. From these figures we see that the best agreement with data is obtained in case '2', though 5 out of the 6 curves pass through all error bars. Nevertheless, in view of the large errors, one should be very cautious in drawing definite conclusions on the precise pole position as well as the preferred OZIF couplings $g_{\rho^0 J/\psi}$ and $g_{\omega J/\psi}$.

Next we show, in Fig. 4, the elastic amplitudes in the $\rho^0 J/\psi$ and $\omega J/\psi$ channels, corresponding to the pole positions 1, 2, 3, i.e., for the smaller values of the OZIF couplings. We see that both amplitudes are very sensitive to the precise pole position, which is logical, as the OZIF channels couple much more weakly to $c\bar{c}$ than $D^0 D^{*0}$, so

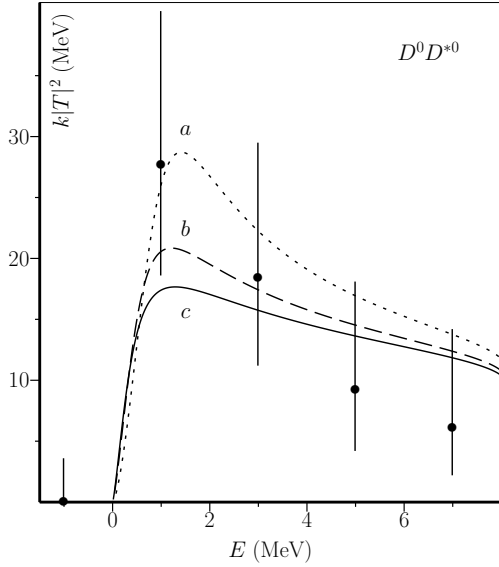


Fig. 3. As in Fig. 2, but now for poles a, b, c.

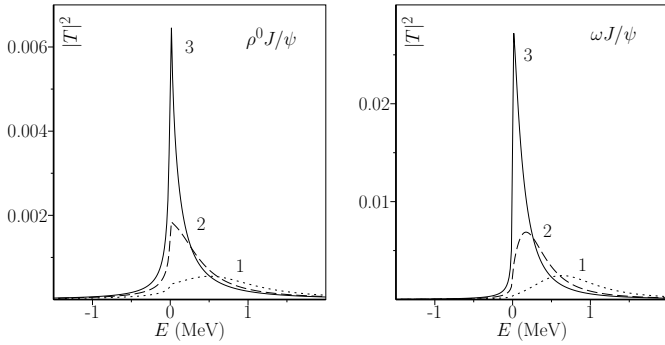


Fig. 4. $\rho^0 J/\psi$ (left) and $\omega J/\psi$ (right) elastic amplitudes for poles 1, 2, 3. Also see Fig. 1 and Table 2.

that the latter channel will strongly deplete the former ones, as soon as it acquires some phase space. This is in line with our analysis in e.g. Ref. [36]. Also note the strongly cusp-like structure of the amplitude in the cases 2 and 3 for $\rho^0 J/\psi$, and 3 for $\omega J/\psi$, which is a manifestation of the depletion due to the opening of the $D^0 D^{*0}$ channel. Such a cusp makes the experimental determination of the $X(3872)$ width very difficult.

In Fig. 5 we take a closer look at the $\omega J/\psi$ and $\rho^0 J/\psi$ amplitudes, in particular how they compare to one another. Now, the effective strength of the $\omega J/\psi$ elastic T -matrix element is 9 times that of $\rho^0 J/\psi$, as its coupling has been chosen 3 times as large (see Table 2 and Eqs. (1–3)). For the corresponding square amplitudes plotted in Fig. 5, this amounts to a factor as large as 81. However, the central $\omega J/\psi$ threshold lies more than 7 MeV above that of $\rho^0 J/\psi$, while the full ω width is only 8.49 MeV. On the other hand, the central $\rho^0 J/\psi$ threshold lies much closer to $D^0 D^{*0}$, while the large physical ρ width strongly boosts the associated amplitude, as demonstrated below.

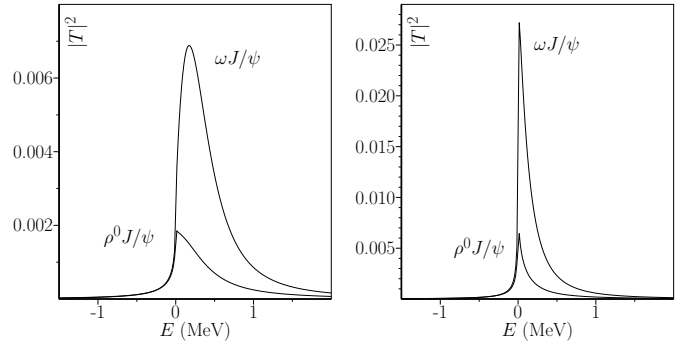


Fig. 5. $\rho^0 J/\psi$ and $\omega J/\psi$ elastic amplitudes, for poles 2 (left) and 3 (right). Also see Fig. 1 and Table 2.

Qualitative arguments in agreement with our calculation were already presented in Ref. [37]. These effects make the maximum $\omega J/\psi$ square amplitude to be only a factor 3.5–4 larger than that of $\rho^0 J/\psi$, both in case 2 and 3, as can be read off from Fig. 5. Moreover, at the precise energy of the respective pole position, the two amplitudes are almost equal in size. Therefore, the observed branching ratio $\mathcal{B}(X(3872) \rightarrow \omega J/\psi)/\mathcal{B}(X(3872) \rightarrow \pi^+ \pi^- J/\psi) \sim 1$ [9,10] is compatible with the present model calculation.

Finally, in order to study the effect of using a complex mass for the ρ^0 in the $\rho^0 J/\psi$ channel, we vary the ρ width from 0% to 100% of its PDG [11] value and plot the corresponding amplitudes in Fig. 6. We see that the maximum $|T|^2$ increases by almost 3 orders of magnitude when going from the 0% case (dotted curve in left-hand plot) to the 100% case (solid curve in right-hand plot). Furthermore, the 0% curve only starts out at the central $\rho^0 J/\psi$ threshold, of course. Thus, it becomes clear that no realistic description of the $\rho^0 J/\psi$ channel is possible without smearing out somehow its threshold, so that its influence kicks in before the $D^0 D^{*0}$ channel opens and depletes the signal. Naturally, similar conclusions apply in principle to the $\omega J/\psi$ channel, though there the effects are less pronounced because of the small ω width and the somewhat higher threshold. These results show that our unitarisation procedure for complex masses in the asymptotic states performs as expected in accounting for thresholds involving resonances.

To conclude our discussion, we should mention that our results are qualitatively in agreement with those of Danilkin & Simonov [20], in the sense that a single resonance pole originating from the $2^3 P_1$ $c\bar{c}$ state is capable of describing the $X(3872)$ data. However, we disagree with their conclusions on the $2^3 P_0$ state. In an earlier, single-channel description [38], we found a resonance at 3946 MeV with a width of 58 MeV, and we do not believe a detailed multichannel calculation will change these values dramatically. Thus, the listed $X(3945)$ [11] resonance, with mass 3916 MeV, width 40 MeV, and positive C -parity, appears to be a good candidate. As for the $2^1 P_1$ state, the $X(3940)$ [11] resonance, with mass 3942 MeV, width 37 MeV, and principal decay mode DD^* , seems the obvious choice. With the old $Z(3930)$ meanwhile identified

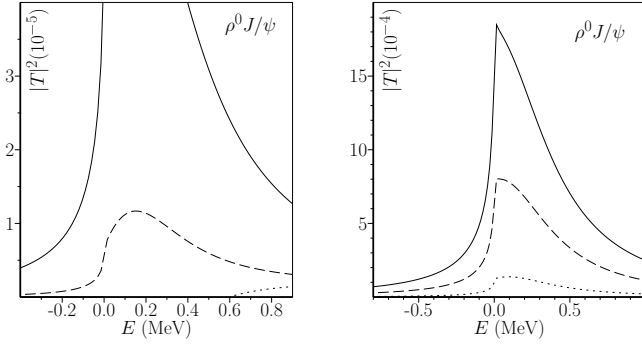


Fig. 6. $\rho^0 J/\psi$ elastic amplitude for reduced ρ^0 width. Left: 0% (dots), 1% (dashes), 5% (full); right: 10% (dots), 50% (dashes), 100% (full). Studied case: pole 2 in Table 2. Also see Fig. 1.

as the 2^3P_2 ($\chi_{c2}(2P)$ [11]) state, we might so understand all 4 charmonium states in the range 3.87–3.95 MeV.

4 Summary and conclusions

Summarising, we have investigated the 1^{++} charmonium scenario for the $X(3872)$ resonance, by analyzing in detail the influence of the $D^0 D^{*0}$, $\rho^0 J/\psi$, and $\omega J/\psi$ channels on pole positions and amplitudes. In order to describe the latter OZIF channels in a realistic way, we have used complex masses for the ρ^0 and ω , and then restored unitarity of the S -matrix by a new and rigorous algebraic procedure, albeit physically heuristic. It is true that the redefined S -matrix may have some unusual analyticity properties [30], but in our amplitudes no sign was found of any nearby spurious singularities. Moreover, the behaviour of the $\rho^0 J/\psi$ amplitude as a function of the ρ^0 width gives us confidence in our approach. Concretely, we have shown that our scenario is compatible with the $D^0 D^{*0}$ and $\pi^+ \pi^- J/\psi$ data, with a single resonance pole on top of or slightly below the $D^0 D^{*0}$ threshold. Moreover, our treatment of the $\rho^0 J/\psi$ and $\omega J/\psi$ channels has proven compatible with the observed branching ratio of these decays.

Thus, the data do not seem to require a molecular or tetraquark interpretation of the $X(3872)$, also in view of so far unobserved [39] charged partner states. Nevertheless, only further improved measurements and theoretical calculations will in the end allow to draw a definitive conclusion on the scenario preferred by nature.

In conclusion, we must stress that the $X(3872)$, whatever its assignment, is an extraordinary structure, because of its coincidence — to an accuracy of less than 1 MeV — with the central thresholds of the principal decay modes. This circumstance is at the same time a blessing and a curse. To start with the latter, no model can ambition to quantitatively describe the $X(3872)$ with present-day state-of-the-art in strong interactions, while experiment will have an extremely hard time to reduce the bin sizes to less than 1 MeV and simultaneously keep statistics sufficiently high. On the other hand, with a strengthened effort of both theory and experiment, a wealth of knowledge on charmonium spectroscopy and strong decay —

OZI-allowed as well as OZI-forbidden — may be gathered by further studying this fascinating resonance.

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A Multichannel T -matrix

The off-energy-shell RSE T -matrix reads [26, 33]

$$T_{ij}^{(L_i, L_j)}(p_i, p'_j; E) = -2\lambda^2 \sqrt{\mu_i p_i r_i} j_{L_i}^i(p_i r_i) \times \sum_{m=1}^N \mathcal{R}_{im} \{[\mathbb{1} - \Omega \mathcal{R}]^{-1}\}_{mj} j_{L_j}^j(p'_j r_j) \sqrt{\mu_j p'_j r_j}, \quad (1)$$

with the diagonal loop function

$$\Omega_{ij}(k_j) = -2i\lambda^2 \mu_j k_j r_j j_{L_j}^j(k_j r_j) h_{L_j}^{(1)j}(k_j r_j) \delta_{ij}, \quad (2)$$

and the RSE propagator

$$\mathcal{R}_{ij}(E) = \sum_{n=0}^{\infty} \frac{g_{(l_c, n)}^i g_{(l_c, n)}^j}{E - E_n^{(l_c)}}. \quad (3)$$

Here, the RSE propagator contains an infinite tower of s -channel bare $q\bar{q}$ states, corresponding to the spectrum of an, in principle, arbitrary confining potential. Furthermore, λ is an overall coupling, r_i is the decay radius of meson-meson (MM) channel i , $E_n^{(l_c)}$ is the discrete energy of the n -th recurrence in the $q\bar{q}$ channel with angular momentum l_c , $g_{(l_c, n)}^i$ is the corresponding coupling to the i -th MM channel, μ_i the reduced mass for this channel, p_i the off-shell relative momentum, and $j_{L_i}^i(p_i)$ and $h_{L_j}^{(1)j}(k_j r_j)$ the spherical Bessel and Hankel functions of the first kind, respectively. Note that μ_i , p_i , and k_i (the on-energy-shell relative momentum) are defined relativistically (see e.g. Ref. [40], Eqs. (16,17)). Also notice that, for a $J^{PC} = 1^{++} c\bar{c}$ system, there is only one confined channel, viz. with $l_c = 1$.

Bound states and resonances can be found by searching for zeros in the determinant of the matrix $(\mathbb{1} - \Omega \mathcal{R})$ in Eqs. (1–3), employing a complex Newton’s method. On the other hand, elastic and inelastic amplitudes are obtained by taking the on-shell values of the corresponding matrix elements of T .

B Redefining the S -matrix

It is straightforward to show that the S -matrix $S(E) \equiv \mathbb{1} + 2iT(E)$, where $T(E)$ is the on-energy-shell restriction of the multichannel T -matrix in Eqs. (1–3), is unitary and symmetric, when limited to open channels and real energies. However, it is also easy to see that, for complex

masses and so complex relative momenta, the unitarity of S is lost, though not its symmetry. The latter property can be used to redefine the physical S -matrix.

Since S is always a symmetrix matrix, it can be decomposed, via Takagi [41] factorisation, as

$$S = V D V^T, \quad (4)$$

where V is unitary and D is a real nonnegative diagonal matrix. Then we get

$$S^\dagger S = (V^T)^\dagger D V^\dagger V D V^T = (V^T)^\dagger D^2 V^T = U^\dagger D^2 U, \quad (5)$$

where we have defined $U \equiv V^T$, obviously unitary, too. So the diagonal elements of $D = \sqrt{U S^\dagger S U^\dagger}$ are the square roots of the eigenvalues of the positive Hermitian matrix $S^\dagger S$, which are all real and nonnegative. Moreover, since $S = \mathbb{1} + 2iT$ is manifestly nonsingular, the eigenvalues of $S^\dagger S$ are even all nonzero and U is unique. Thus, we may define

$$S' \equiv S U^\dagger D^{-1} U. \quad (6)$$

Then, using Eq. (4) and $V = U^T$, we have

$$S' = U^T D U U^\dagger D^{-1} U = U^T U, \quad (7)$$

which is obviously symmetric and, as

$$(U^T U)^\dagger = U^\dagger (U^\dagger)^T = U^{-1} (U^{-1})^T = (U^T U)^{-1}, \quad (8)$$

also unitary. So S' has the required properties to be defined as the S -matrix for a scattering process with complex masses in the asymptotic states.

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